Modele DSGE

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Modele DSGE

- DSGE - *Dynamic Stochastic General Equilibrium Model*
- Elements of the general equilibrium model:
  - housholds sector
  - firm sector
  - publiczny sector (monetary authority)
- Household sector and firm sector are rational (optimizing behaviour)
- Maximisation of utility and profits:
  - intertemporal
  - under uncertainty
- Monetary authority set the interest rate according to some rule (e.g. Taylor rule), alternatively it is maximising its own objective function

Modele DSGE - solving model and analysis

- Usually obtaining analytical solutions too difficult - so approximations against logarithms are used
- Bring the model to a form approximated by the Error Correction Mechanism (linearization)
- Solve the model by finding expressions for the expected magnitudes of the variables as functions of the variables (rather than their expected values)
- Further analysis involves examining the dynamic values of the model by analyzing the reaction function, docomposition of the forecast error, etc.

State space representation

- It is possible to obtain the following linearized representation of a DSGE model in state space::

$$\mathbf{x}_t = \mathbf{H}\xi_t$$
$$\xi_{t+1} = \mathbf{F}\xi_t + \mathbf{v}_{t+1}$$

where ξ_t is a state vector and $\mathbf{x}_t$ vector of observable endogenous variables

- A model defined in this way is usually singular: there are fewer random shocks than endogenous variables
- Using the Kalman filter it is possible to filter out $\xi_{t|t-1}$
- Using $\xi_{t|t-1}$ the likelihood function for the estimated model can be formulated

State space representation

- The model can be estimated using MNW if the number of structural shocks is equal to the number of endogenous variables
- Usually, however, the number of endogenous variables is greater than the number of structural shocks (singularity)
- Possible solutions:
  - We estimate the model using only a part of observable variables
  - We add random (measurement) errors to the state space representation
- In this case the model will take the form:

$$\mathbf{x}_t = \mathbf{H}\xi_t + \mathbf{u}_t$$
$$\xi_{t+1} = \mathbf{F}\xi_t + \mathbf{v}_{t+1}$$

  assuming, że $\mathbf{u}_t$ i $\mathbf{v}_{t+1}$ are not correlated
- measurement errors have no structural interpretation

Bayesian estimation

- It often happens that we have some a priori knowledge about the parameter vector θ
- In such a case, we can use the Bayesian approach to improve the precision of the estimates
- Bayes' theorem implies that

$$f\left(\theta \middle| \mathbf{X}\right) = \frac{f\left(\mathbf{X} \middle| \theta\right) f\left(\theta\right)}{f\left(\mathbf{X}\right)},$$

where $f\left(\theta \middle| \mathbf{X}\right)$ is posterior dansity, $f\left(\theta\right)$ a priori density, $f\left(\mathbf{X} \middle| \theta\right)$ likelihood function, $f\left(\mathbf{X}\right)$ unconditional density function of the observed sample

Bayesian estimators: point estimation

- A popular way to obtain point estimates using Bayesian methods is to use the modal value $f\left(\theta|\,\mathbf{X}\right)$
- Note: in the special case of the normal distribution, the modal value and the mean are equal
- From the definition of the modal value and the monotonicity of the logarithm, it follows that

$$\max_{\theta} \ln f\left(\theta|\,\mathbf{X}\right) = \max\left[\ln f\left(\mathbf{X}|\,\theta\right) + \ln f\left(\theta\right)\right]$$

- This formula is equivalent to the formula for the ML estimator for the likelihood function

$$\ell\left(\theta\right) = \ln f\left(\mathbf{X}|\,\theta\right) + \ln f\left(\theta\right)$$

- In the case of the normal distribution, the prior distribution is usually assumed to have the form $\theta \sim N\left(0, \Sigma\right)$
- The matrix Σ represents the researcher's uncertainty about his a priori knowledge